Chapter 14: Wave Motion
Tuesday April 7th

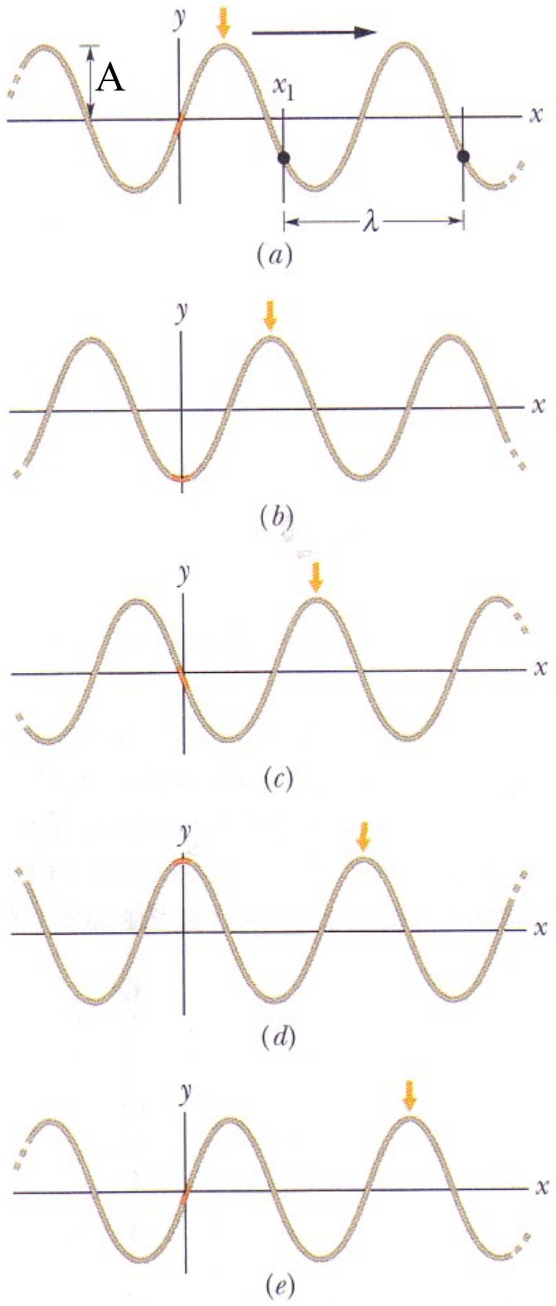
- Wave superposition
  - Spatial interference
  - Temporal interference (beating)
  - Standing waves and resonance
  - Sources of musical sound
- Doppler effect
  - Sonic boom
- Examples, demonstrations and *i*clicker

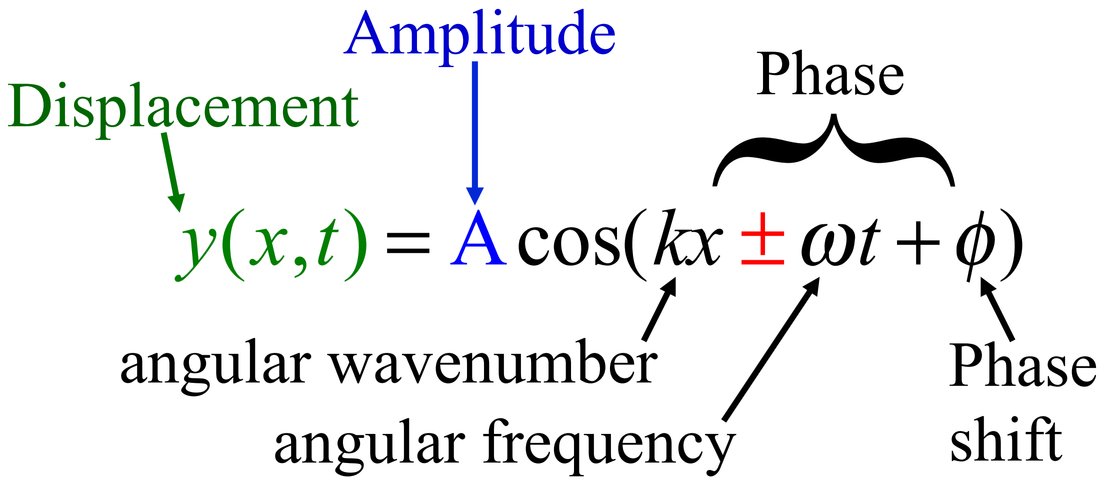
- Final 25 minute Mini Exam on Thursday
- Will cover oscillations and waves (LONCAPA #'s 18-21)

Reading: up to page 242 in Ch. 14

Review - wavelength and frequency

Transverse wave




Amplitude

Displacement

Phase

$$y(x,t) = A\cos(\overbrace{kx \pm \omega t + \phi})$$

angular wavenumber

angular frequency

Phase shift

$$k = \frac{2\pi}{\lambda}$$

k is the angular wavenumber,
λ is the wavelength.

$$\omega = \frac{2\pi}{T}$$

ω is the angular frequency.

frequency $f = \frac{1}{T} = \frac{\omega}{2\pi}$

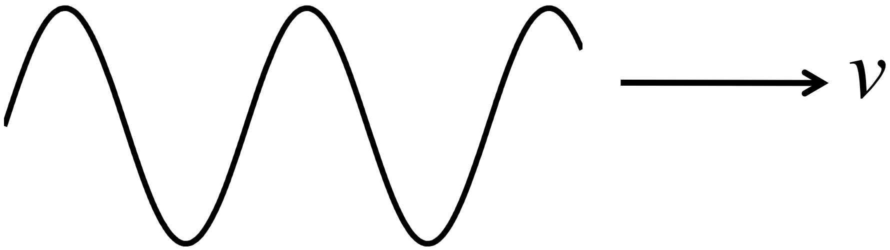
velocity $v = \mp\frac{\omega}{k} = \mp\frac{\lambda}{T} = \mp f\lambda$

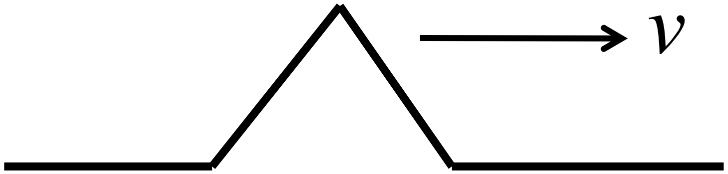
The wave equation

$$\frac{F_T}{\mu}\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

- General solution for transverse waves on a tensioned string:

$$y(x,t) = \mathrm{A}\sin\left(kx \pm \omega t\right) \quad \text{or} \quad y(x,t) = \mathrm{A}\times f^{n}\left(kx \pm \omega t\right)$$





$$\frac{\partial^2 y}{\partial x^2} = -k^2 y\left(x,t\right) \qquad \frac{\partial^2 y}{\partial t^2} = -\omega^2 y\left(x,t\right)$$

$$-\frac{F_T}{\mu}k^2 = -\omega^2 \quad \text{or} \quad \frac{\omega^2}{k^2} = v^2 = \frac{F_T}{\mu}$$

$$\boxed{\Rightarrow \quad v = \sqrt{\frac{F_T}{\mu}}}$$

The principle of superposition for waves

- If two waves travel simultaneously along the same stretched string, the resultant displacement y' of the string is simply given by the summation

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

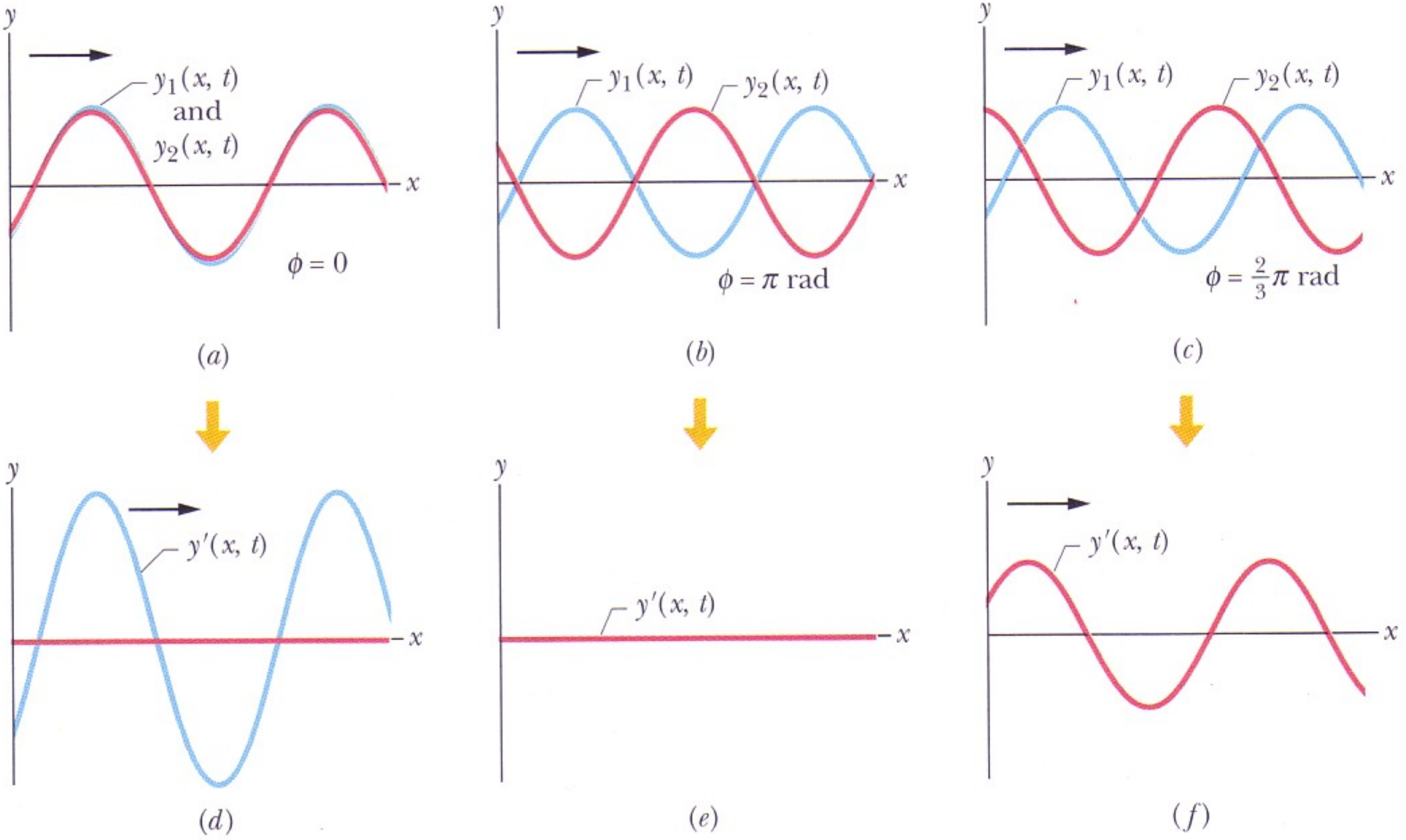
where y_1 and y_2 would have been the displacements had the waves traveled alone.

- This is the principle of superposition.

Overlapping waves algebraically add to produce a **resultant wave** (or **net wave**).

Overlapping waves do not in any way alter the travel of each other

Interference of waves



Interference of waves

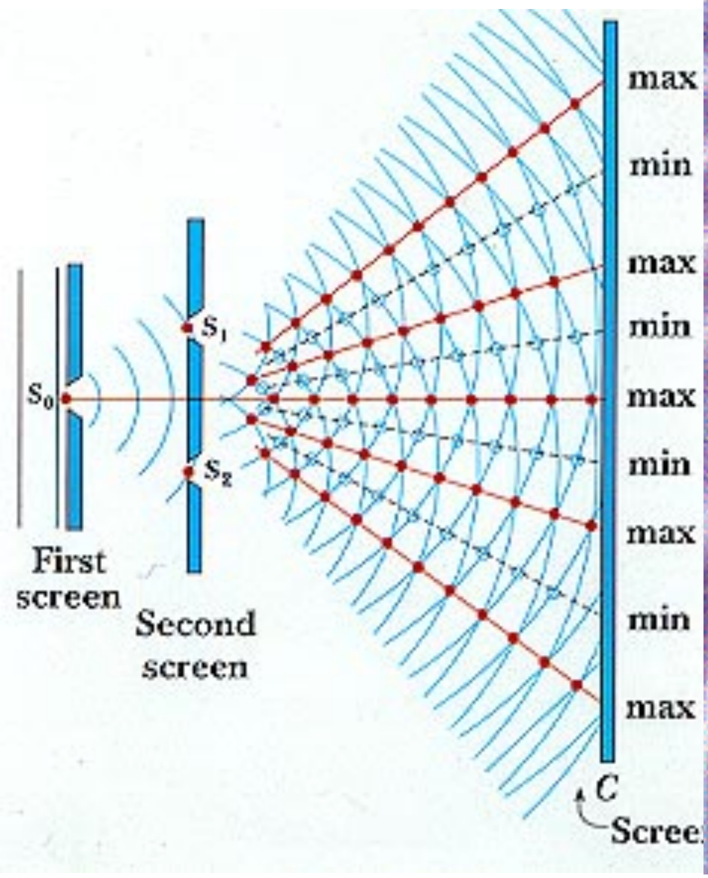
$$y'(x,t)=\left[2\mathrm{A}\cos\tfrac{1}{2}\phi\right]\sin\left(kx-\omega t+\tfrac{1}{2}\phi\right)$$

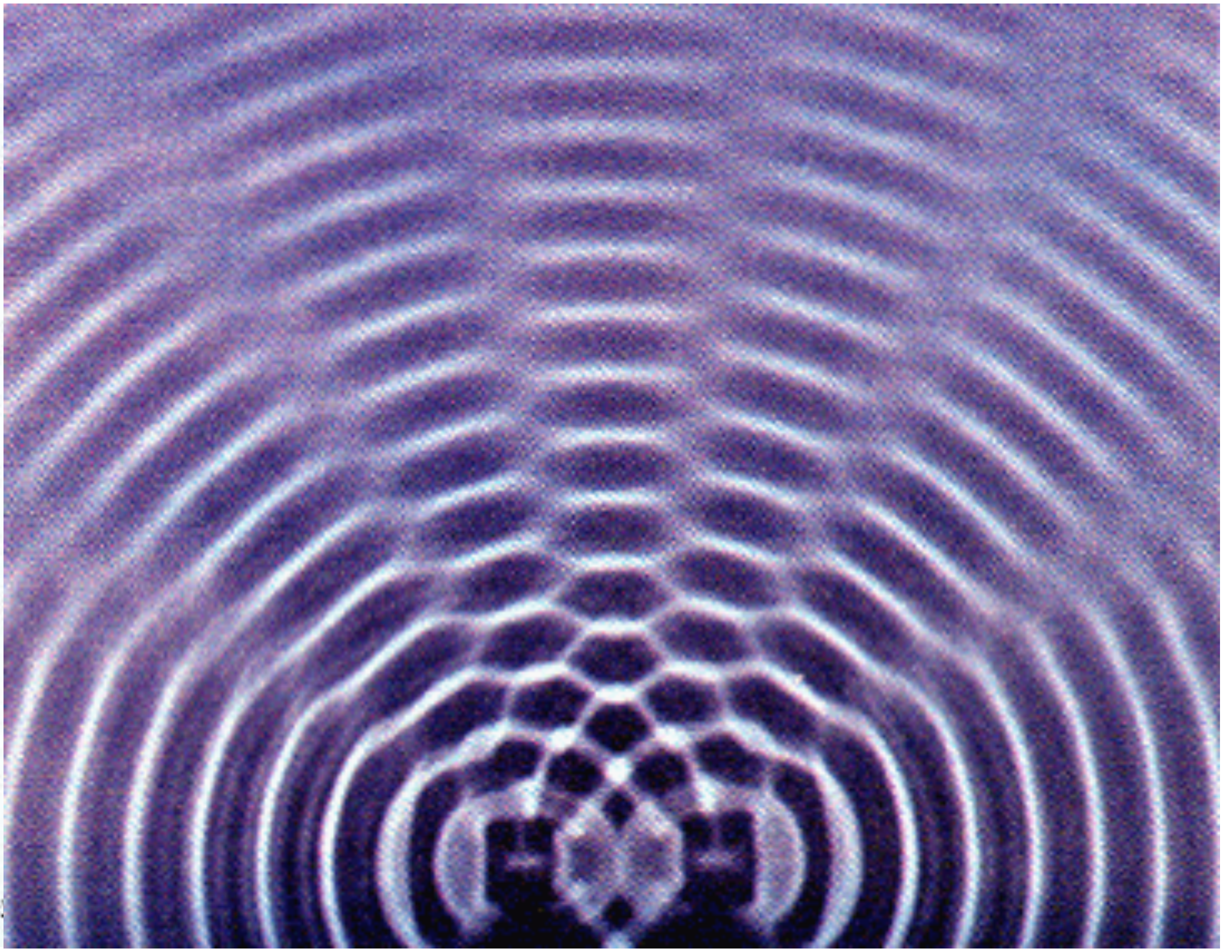
If two sinusoidal waves of the same amplitude and frequency travel in the same direction along a stretched string, they interfere to produce a resultant sinusoidal wave traveling in the same direction.

- If $\varphi = 0$, the waves interfere constructively, $\cos\frac{1}{2}\varphi = 1$ and the wave amplitude is $2\mathrm{A}$.
- If $\varphi = \pi$, the waves interfere destructively, $\cos(\pi/2) = 0$ and the wave amplitude is 0, *i.e.*, no wave at all.
- All other cases are intermediate between an amplitude of 0 and $2\mathrm{A}$.
- Note that the phase of the resultant wave also depends on the phase difference.

Adding waves as vectors (phasors) described by amplitude and phase

Wave interference - spatial





Wave interference - temporal beating

- Suppose:

$$y_1 = \mathrm{A}\sin\left(kx-\omega t\right)$$

$$y_2 = \mathrm{A}\sin\left\{kx-\left(\omega+\delta\right)t\right\}$$

δ represents a tiny difference in frequency between the two waves

Then:

$$y'\left(x,t\right) = y_1\left(x,t\right)+y_2\left(x,t\right)$$

$$= \mathrm{A}\sin\left(kx-\omega t\right)+\mathrm{A}\sin\left(kx-\omega t-\delta t\right)$$

But:

$$\sin\alpha+\sin\beta = 2\cos\tfrac{1}{2}\left(\alpha-\beta\right)\sin\tfrac{1}{2}\left(\alpha+\beta\right)$$

So:

$$y'\left(x,t\right) = \underbrace{\left[2\mathrm{A}\cos\tfrac{1}{2}\delta t\right]}_{\text{Slowly varying Amplitude}}\underbrace{\sin\left\{kx-\left(\overbrace{\omega+\tfrac{1}{2}\delta}^{\text{Average frequency}}\right)t\right\}}_{\text{Original wave part}}$$

Interference - Standing Waves

If two sinusoidal waves of the same amplitude and wavelength travel in **opposite** directions along a stretched string, their interference with each other produces a **standing wave**.

$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

$$= A\sin(kx-\omega t) + A\sin(kx+\omega t+\phi)$$

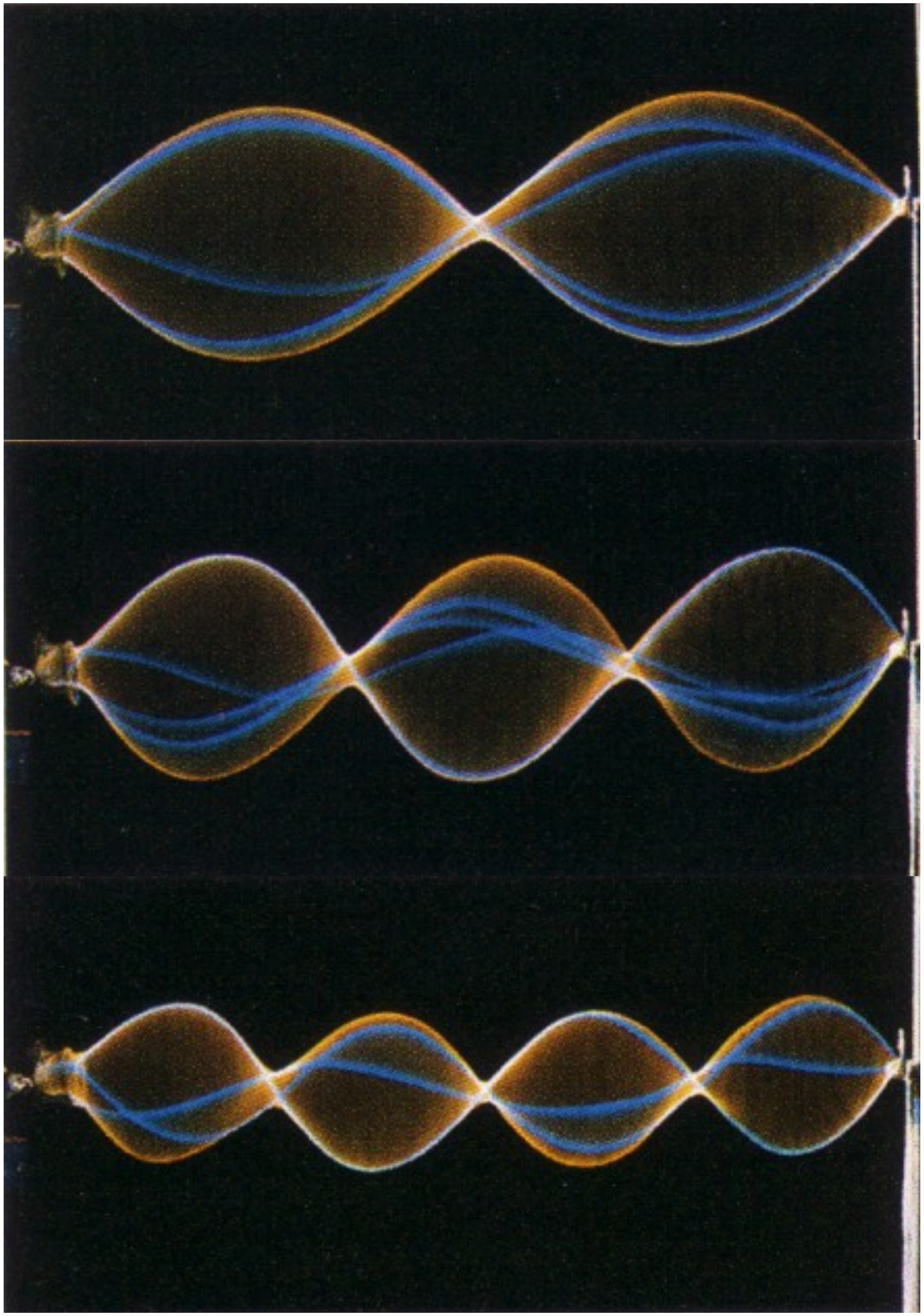
$$= 2A\underbrace{\cos\left(\omega t+\tfrac{1}{2}\phi\right)}_{t\ \textbf{dependence}}\underbrace{\sin\left(kx+\tfrac{1}{2}\phi\right)}_{x\ \textbf{dependence}}$$

- This is clearly not a traveling wave, because it does not have the form $f^n(kx-\omega t)$.
- In fact, it is a stationary wave, with a sinusoidal varying amplitude $2A\cos(\omega t)$.

Link

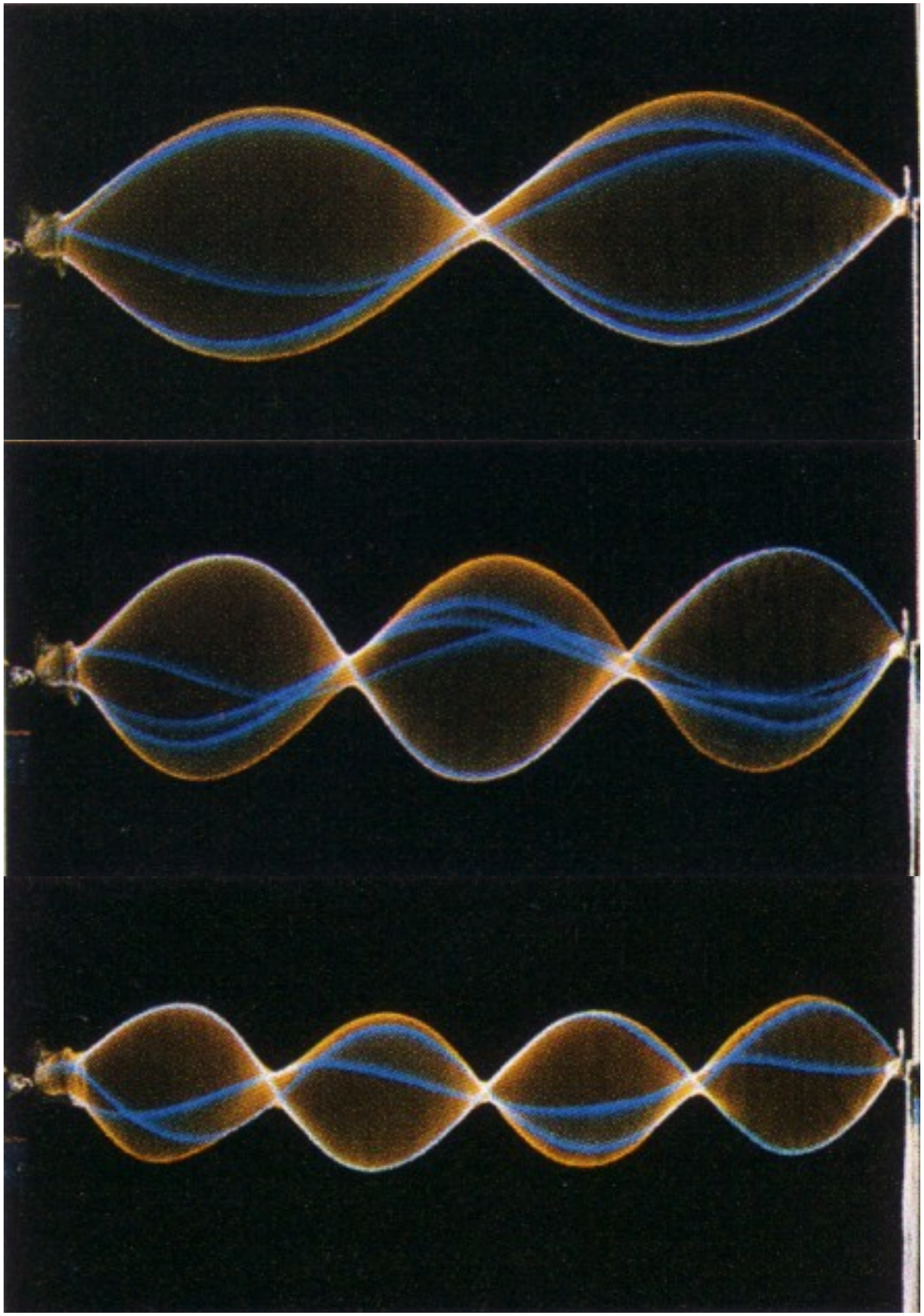
Standing waves and resonance

- At ordinary frequencies, waves travel backwards and forwards along the string.
- Each new reflected wave has a new phase.
- The interference is basically a mess, and no significant oscillations build up.

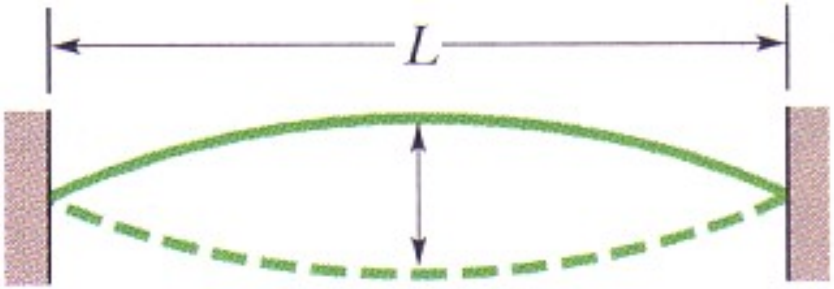


Standing waves and resonance

- However, at certain special frequencies, the interference produces strong standing wave patterns.
- Such a standing wave is said to be produced at resonance.
- These frequencies are called resonant frequencies.

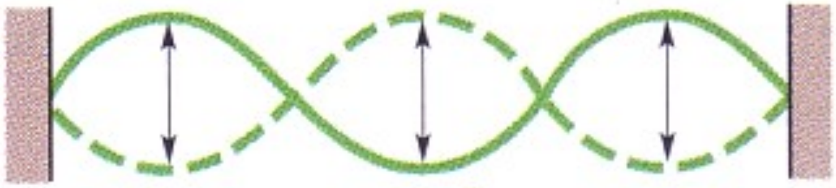


Standing waves and resonance



$L = \frac{\lambda}{2}$

$L = \lambda = \frac{2\lambda}{2}$



$L = \frac{3\lambda}{2}$

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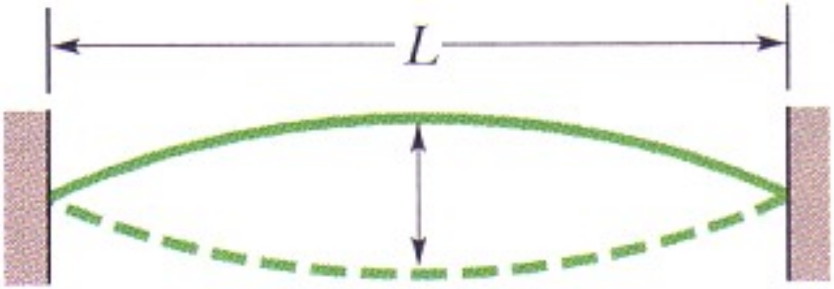
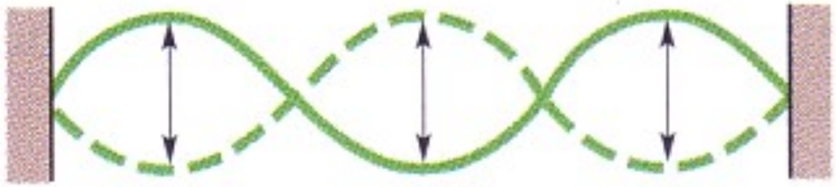
Standing waves and resonance



$L = \frac{\lambda}{2}$

λ determined by geometry

$L = \lambda = \frac{2\lambda}{2}$



$L = \frac{3\lambda}{2}$

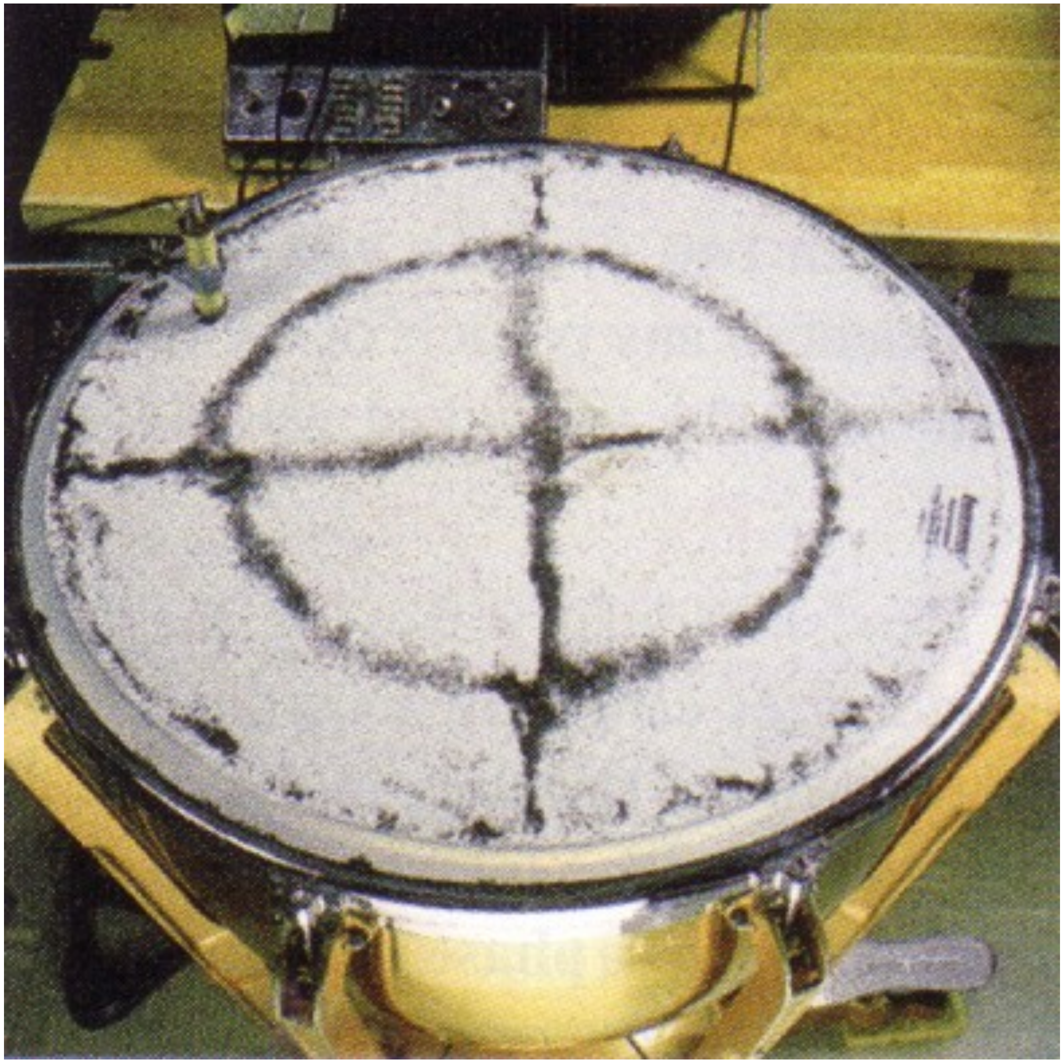
- Standing waves occur whenever the phase of the wave returning to the oscillating end of the string is precisely in phase with the forced oscillations.
- Thus, the trip along the string and back should be equal to an integral number of wavelengths, *i.e.*

$$2L = n\lambda \quad \text{or} \quad \lambda = \frac{2L}{n} \quad \text{for } n = 1,2,3...$$

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \quad \text{for } n = 1,2,3...$$

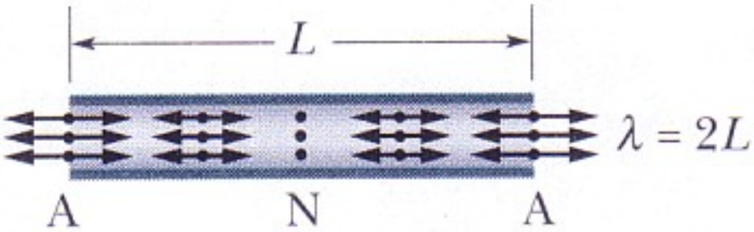
- Each of the frequencies f_1, f_1, f_1, *etc*, are called harmonics, or a harmonic series; n is the harmonic number.

Standing waves and resonance



- Here is an example of a two-dimensional vibrating diaphragm.
- The dark powder shows the positions of the nodes in the vibration.

Standing waves in air columns



(a)

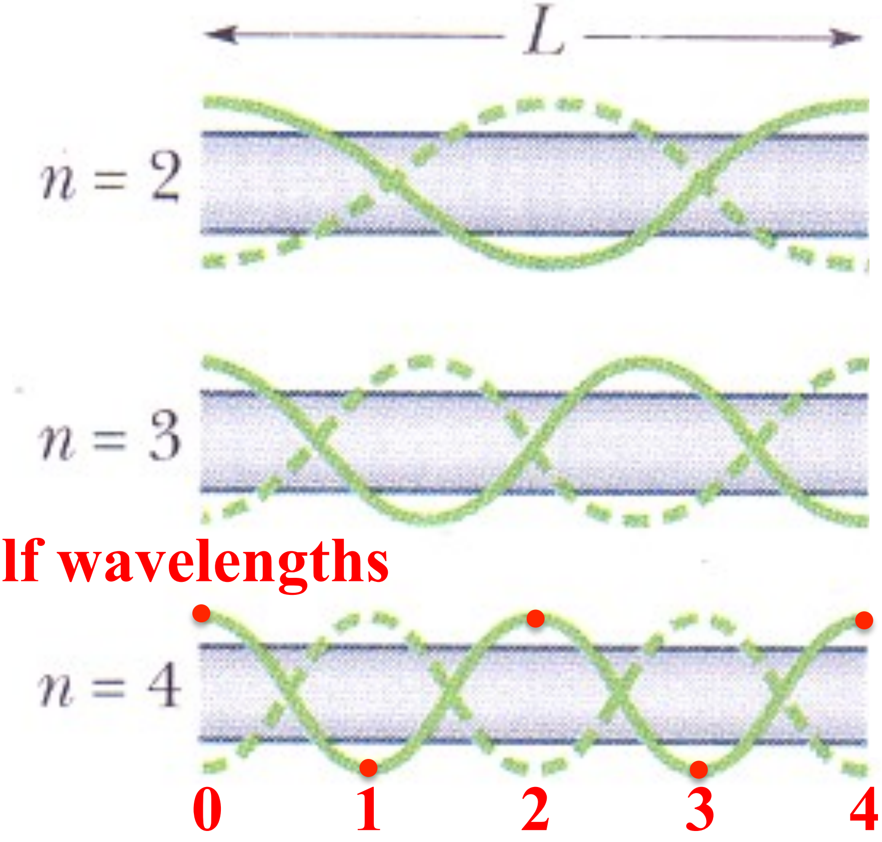
- Simplest case:
  - 2 open ends
  - Antinode at each end
  - 1 node in the middle
- Although the wave is longitudinal, we can represent it schematically by the solid and dashed green curves.

$$L = \tfrac{1}{2}\lambda$$

$$\Rightarrow \lambda = 2L = 2L/1$$

Standing waves in air columns

A harmonic series



$L = \tfrac{1}{2}\lambda$ $\qquad \lambda = 2L/1$

$L = \lambda = \tfrac{2}{2}\lambda$ $\qquad \lambda = 2L/2$

$L = \tfrac{3}{2}\lambda$ $\qquad \lambda = 2L/3$

$L = \tfrac{4}{2}\lambda$ $\qquad \lambda = 2L/4$

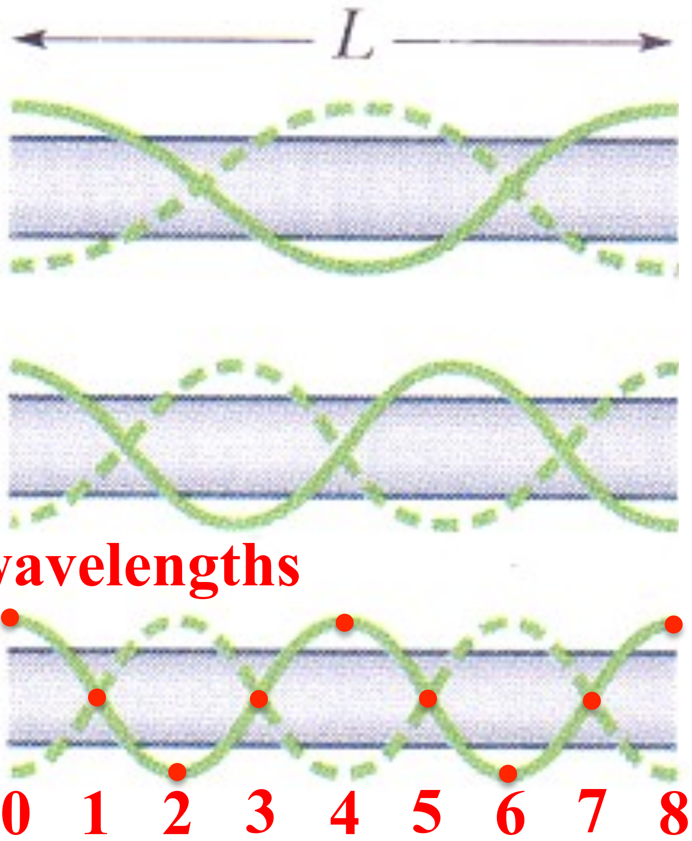
$$\lambda = \frac{2L}{n}, \quad \text{for } n = 1,2,3,\ldots.$$

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \qquad \text{for } n = 1,2,3,\ldots$$

Standing waves in air columns

A harmonic series

$L = \frac{2}{4}\lambda$ $\quad \lambda = 4L/2$



$n = 4$ $\quad L = \frac{4}{4}\lambda$ $\quad \lambda = 4L/4$

$n = 6$ $\quad L = \frac{6}{4}\lambda$ $\quad \lambda = 4L/6$

Quarter wavelengths

$n = 8$ $\quad L = \frac{8}{4}\lambda$ $\quad \lambda = 4L/8$

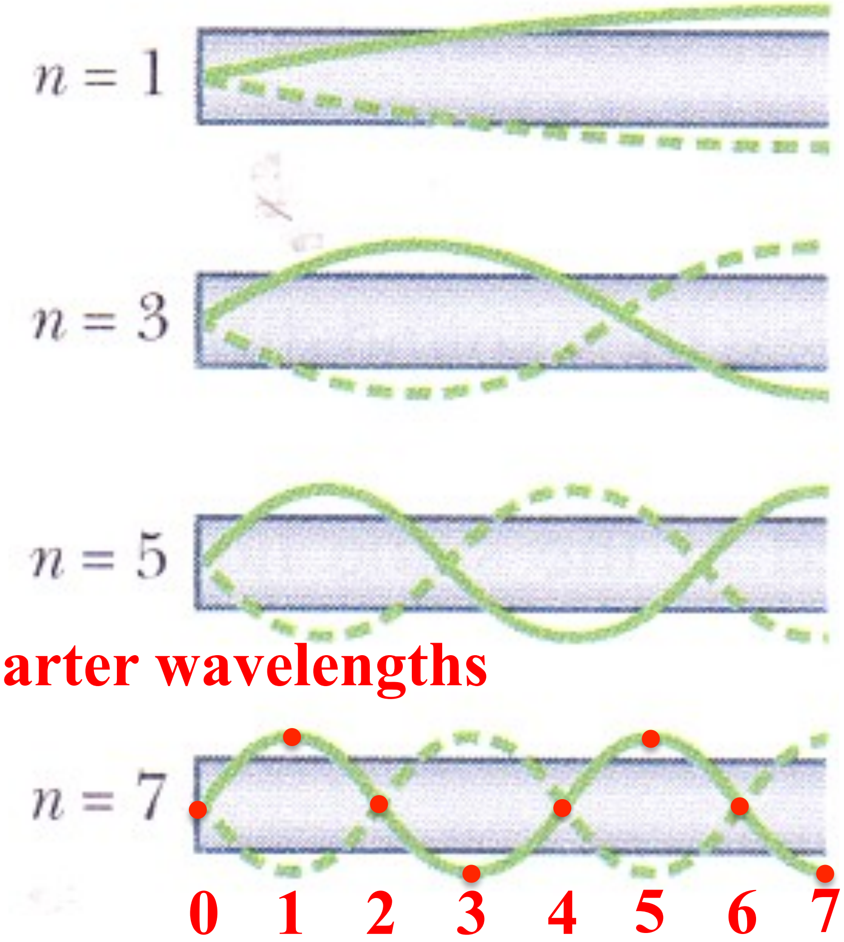
$$\lambda = \frac{4L}{n}, \quad \text{for } n = 2,4,6,....$$

$$f = \frac{v}{\lambda} = n\frac{v}{4L}, \qquad \text{for } n = 2,4,6,...$$

WARNING: This slide is not in the textbook!

Standing waves in air columns

A different harmonic series



Quarter wavelengths

$$L = \tfrac{1}{4}\lambda \qquad \lambda = 4L/1$$

$$L = \tfrac{3}{4}\lambda \qquad \lambda = 4L/3$$

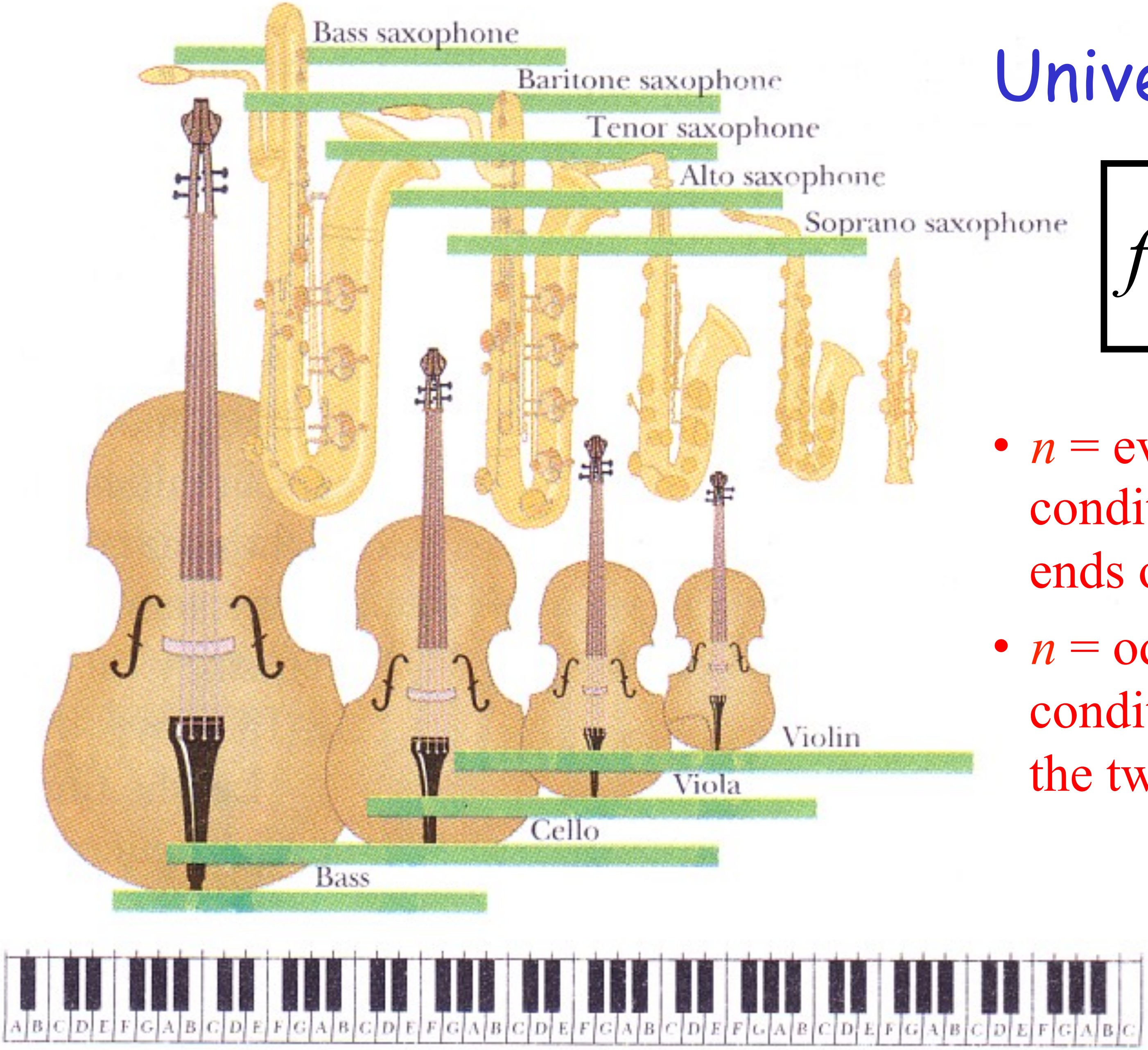
$$L = \tfrac{5}{4}\lambda \qquad \lambda = 4L/5$$

$$L = \tfrac{7}{4}\lambda \qquad \lambda = 4L/7$$

$$\lambda = \frac{4L}{n}, \quad \text{for } n = 1,3,5,\ldots.$$

$$f = \frac{v}{\lambda} = n\frac{v}{4L}, \qquad \text{for } n = 1,3,5,\ldots$$

Musical instruments



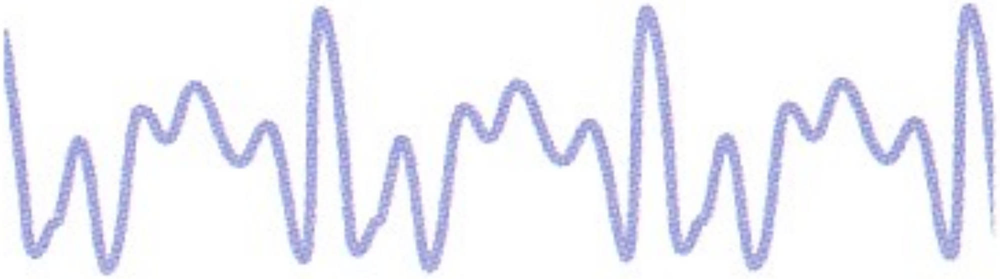
Universal Result

$$\boxed{f = \frac{v}{4L} \times n}$$

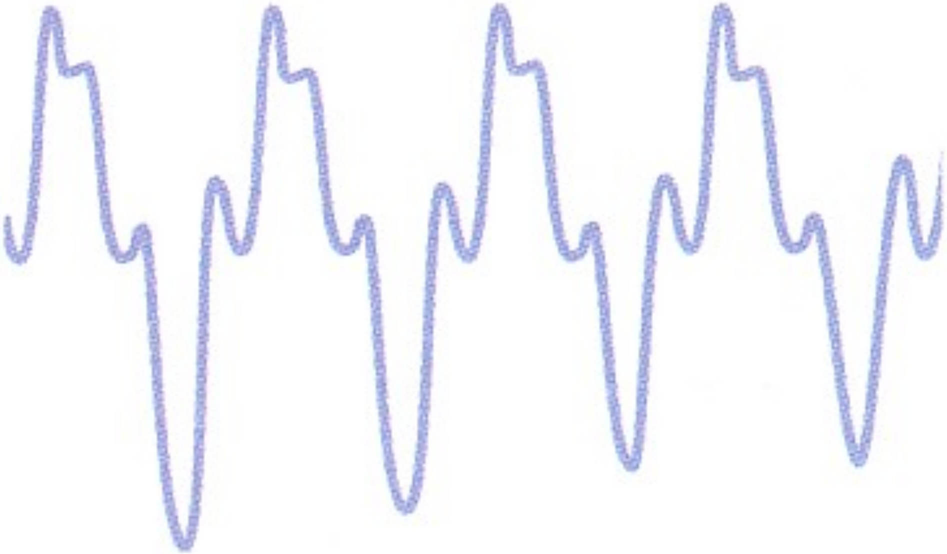
- n = even if boundary condition same at both ends of pipe/string
- n = odd if boundary condition different at the two ends

Musical instruments

Flute



Oboe



Saxophone

Link

Doppler effect

- Consider a source of sound emitting at a 'proper frequency' , f , moving relative to a stationary observer.
- The observer will hear the sound with an apparent frequency, f', which is shifted from the proper frequency according to the following Doppler equation:

$$f' = \frac{f}{\left(1 \pm u/v\right)}$$

- Here, v is the sound velocity (~330 m/s in air), and u is the relative speed between the source and detector.

When the source is moving towards the observer, use the minus (-) sign so that the formula gives an upward shift in frequency. When the source is moving away from the observer, use the plus (+) sign so that the formula gives a downward shift in frequency.

Mach cone angle: $$\sin\theta = \frac{v}{v_S}$$

Doppler effect

- **Now consider a moving observer and a stationary source.**
- The source again emits at the 'proper frequency', f.
- The observer will hear the sound with an apparent frequency, f', which is shifted from the proper frequency according to the following Doppler equation:

$$f' = \left(1 \pm \frac{u}{v}\right) f \qquad ^{**}$$

This time, use the plus (+) sign if the observer is moving towards the source, so that the you again get an upward shift in frequency. Use the minus (−) sign when the observer is moving away from the source, so that the formula gives a downward shift in frequency.

**The moving source/observer equations become equivalent when $u << v$. In such cases, use your intuition to pick the sign: approaching, f' must increase; receding, f' must decrease.